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The use of some historical mathematical textbooks from the Teachers' Institute in Spišská Kapitula in the 19th century and first half of 20th century

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Abstract

We would like to present some excerpts of historical mathematical textbooks from the Teachers' Institute in Spišská Kapitula, because this institute is at the roots of primary education in Central Europe. The methodological approaches used in these textbooks contain suitable motivation and models for explaining basic mathematical notions in primary education and the training of future primary teachers. There are some projects in Europe which use educational software and modern modes of interpretation to present sections from historical mathematical textbooks in contemporary teaching. In our article we would like to do the same for the Slovakian situation.

Keywords: textbook, history of education, teachers institute, mathematics, curriculum

Introduction

In the 1819/1820 school year, the first Teachers' Institute was founded in Spišská Kapitula in the building of a priests' seminary. It was the first specialised school for teachers in Ugria (Slovakia was at that time part of Ugria). Juraj Paleš (1753-1833), the Spiš canonist, was the first headmaster. The school existed until the year 1949, when its activities were violently interrupted.

The textbook, *Pedagogy for Primary Schools of Spiš*, written by Juraj Paleš in 1820, was published in Levoča for the teacher candidates and students of pedagogy. In our article, we describe the structure of this textbook. Since the textbook is similar to Lesnyánszky András' (1795-1859) textbook, *Didactics and Methodology*, written in 1832, we present some mathematical material from both textbooks.

Franz Močnik (1814-1892) was the most important author of mathematical textbooks in the Austrian-Ugrian monarchy in the second half of the 19th century. Originally, his texts were written in German, but they were subsequently translated into 14 other languages. They were used also in the Teachers' Institute in Spišská Kapitula.

In addition, during the period of 1918-1939, some of the mathematics textbooks used were translated from the Czech language into the Slovak language and we present part of the contents of these textbooks.

1. Creation of the Teachers' Institute in Spišská Kapitula

Early in the 19th century, Johann Ladislaus Pyrker (1772-1847) became the bishop of Spiš. After being named Bishop on August 18th 1818, he came to his diocese as a new bishop on May 12th 1819, and on 21st July 1819, he made his canonical visitation trip.

The result of this trip was that he became aware of the low teaching level of teachers at the schools in the parishes and villages. This can be seen in his letter from August 24th 1819 to the Austrian emperor:

> The situation in the schools is as follows: there is a high level at the secondary schools, but the village schools do not have enough able teachers. I propose for this reason that by secondary school in Levoča, a preparatory school for teachers in the villages be created. The students who would like to become teachers at village schools should have some hours of instruction in educational methodology and in organ playing. (Pyrker 1966)

Later, Bishop Pyrker changed the proposed location for the Teachers' Institute (see Gejdoš 2007). His argumentation can be seen in the following letter extract:

> I think that the young men who study in Levoča and would like to be teachers in a village will have little time for the preparatory institute alongside their other school subjects, and if they live in private houses without the needed guidance, then they will not receive the training for the expected goal... For this reason it will be better to create the teachers' institute not in Levoča, but here in Spišská Kapitula. For the teachers' institute, a suitable building will be the priests' seminary...

Juraj Páleš (1753-1833), a dignitary of the cathedral, became the first director of the Teachers' Institute in Spišská Kapitula. Its aim was to produce qualified teachers for village schools. In the State Archives in Levoča, in the collection of the Office of the Bishop in Spišská Kapitula, there are two letters by Pyrker to Páleš (see Olejník 2007).

2. The pedagogy textbook by Juraj Páleš – the first in the Slovak language

Juraj Páleš did not only administer and organise the Institute, but he also taught. As a teacher he wrote two textbooks for his students. The first one is *Obradoslovie* [Liturgics], a textbook for organists written in Latin. The second textbook is *Pedagogy for Primary Schools of Spiš* (see Páleš 1820).

Juraj Páleš wrote it in Latin, but later translated the book into Slovak for those readers who did not master Latin. In the period 1776-1805 he was in the villages surrounding Spišská Kapitula. During this time when he observed that the level of education in the village schools was low, he took part in the creation of these schools and he also taught at these schools. The information and experience he gained in these activities was published in his textbook. At that time, Juraj Páleš' was the first textbook on pedagogy in Hungary and, what is more, it was written in Slovak. The Slovak language used in the book follows the conventions of the first Slovak grammar, prepared in 1787 by Anton Bernolák[1].

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[1] Anton Bernolák (1762 - 1813) was a Slovak priest, who codified the first version of Slovak language on the base of west Slovak dialect. Nowadays the Slovak language is based on the codification by Ľudovít Štúr (1815 - 1856), who codified the language in the year 1843 on the base of central Slovak dialect.

PÆDAGOGIA
Slowenſká
pre
Triwiálſké Školi
Biſkúpſtwa Spiſſkého,
ſpiſaná
ſkrz
G. P.
M. DCCC. XX.

W Lewoči
witlačená u Jána Werthmüllera.

Figure 1. Title page of the textbook “*PEDAGOGY*”

The textbook contains from four main parts:

1. The teaching process in folk (normal) schools;
2. The curriculum for the second year in folk (normal) school;
3. The curriculum for the third year in folk (normal) school;
4. Recommendations for the teachers of the Spiš diocese.

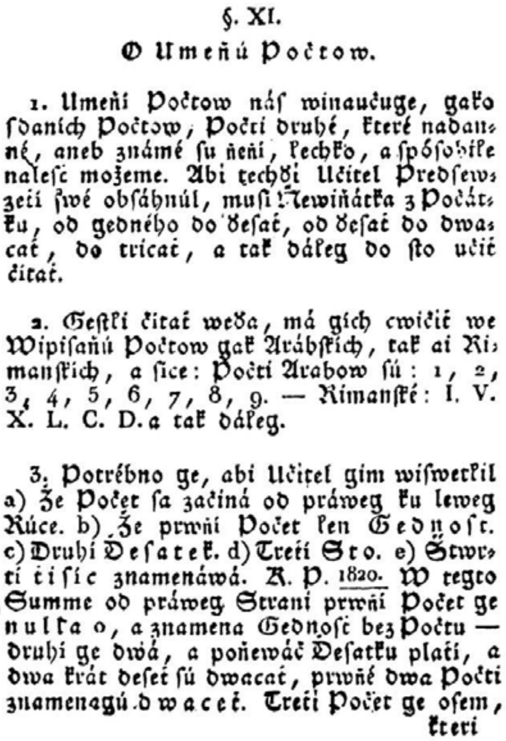
In the first part there are paragraphs about teaching methods in folk schools, and individual and group work. The first section also describes general rules for writing, reading, and catechetical instruction. A paragraph about curriculum for the first year in folk school is also contained in this section. In the second part there are paragraphs about Christian teaching, moral exercises, and grammar. In the third part there are new paragraphs about biblical history, Latin, and the Hungarian language.

In the fourth part, the first paragraph introduces some information about the structure of school administration in Ugria. Juraj Páleš wrote here about five regional directors for school administration, located in: Bratislava, Košice, Pécs, Oradea and Zagreb. He explains that the folk schools in the historical Orava and Liptov regions belong to the director in Bratislava, and the schools in the Spiš region to the director in Košice. In the next paragraphs there are recommendations for teachers on how to start, lead and finish teaching in the school and what the teacher should do after teaching.

In every section there is the paragraph about the teaching of arithmetic, which has very interesting examples that are still useful nowadays in mathematics education at the primary level.

3. Mathematics education in the textbook *Pedagogy*

In his textbook, Juraj Páleš deals with questions of mathematics education for the primary level on approximately 6 pages. Here, we present some excerpts from the textbook with the translation from Slovak to English, starting on page 22, Section XI:



§. XI.

O Umeňú Počtow.

1. Umeňi Počtow náſ winaučuge, gako ſdanich Počtow, Počti druhé, které nadanné, aneb známé ſu ňeňi, lechko, a ſpóſobile naleſť možeme. Abi techdi Učitel Predſewzeťi ſwé obſáhnúl, muſi Newiňátka z Počátku, od gedného do deſať, od deſať do dwacať, do tricať, a tak dáleg do ſto učiť čitať.

2. Geſtli čitať weda, má gich cwičiť we Wipiſaňú Počtow gak Arábſkich, tak ai Rimanſkich, a ſice: Počti Arabow ſú: 1, 2, 3, 4, 5, 6, 7, 8, 9. — Rimanſké: I. V. X. L. C. D. a tak dáleg.

3. Potrébno ge, abi Učitel gim wiſwetlil a) Že Počet ſa začíná od práweg ku leweg Rúce. b) Že prwňi Počet len Gednoſť. c) Druhi Deſatek. d) Treťi Sto. e) Štwrtí tiſic znamenáwá. N. P. 1820. W tegto Summe od práweg Strani prwňi Počet ge nulka 0, a znamena Gednoſť bez Počtu — druhi ge dwá, a poňewáč Deſatku plaťi, a dwa krát deſeť ſú dwacať, prwňé dwa Počti znamenagú dwaceť. Treťi Počet ge oſem, kteri

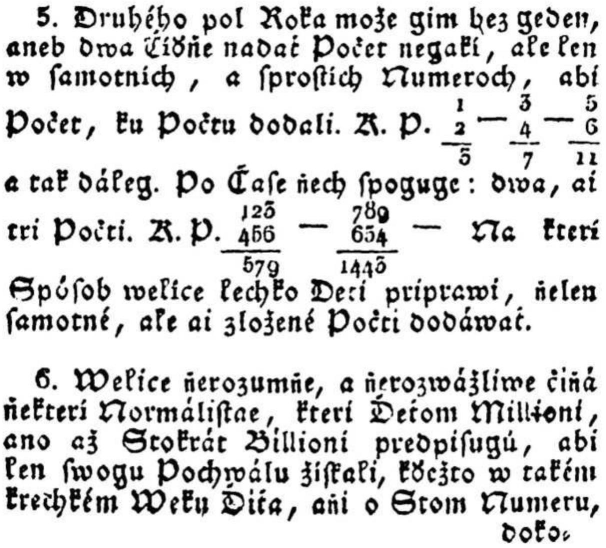
Translation:

> *1. The art of figures teaches us how to easily and capably find, from given examples, other examples that are unknown. To reach this goal a teacher has to, from the very beginning, teach innocents how to count from one to ten, from ten to twenty, to thirty etc., to one hundred.*
>
> *2. When they can count, he should train them in writing both Arabic and Roman figures, i.e. Arabic figures: 1, 2, 3, 4, 5, 6, 7, 8, 9. – Roman: I. V. X. L. C. D. etc.*
>
> *3. It is necessary for the teacher to explain:*
>
> *a) That counting starts from the right hand to the left hand. b) That the first figure only consists of one digit – number one. c) The second figure consists of tens. d) The third figure consists of hundreds. e) The fourth figure consists of thousands, e.g. 1820. In this result from the right hand the first figure is zero 0 and it represents units without any counting – the second one is two and that is why it represents tens, because two times ten is twenty. The first two numbers mean twenty. The third number is eight. This figure stands for hundreds because eight times*

> *one hundred is eight hundred. So the first, second and third figure stand for the sum of eight hundred and twenty. However, when we add the fourth number one, this number stands for thousands, so the final sum represents the year one thousand eight hundred and twenty.*

Figure 2. Excerpt from *Pedagogy* (p. 22)

This text, on page 23, contains some recommendations for a teacher in the first grade of primary school. The topic pertains to the second semester (half-year). In this grade, teaching of counting operations is expected, as we can see below (addition operation):



5. Druhého pol Roka može gim bez geden, aneb dwa Tídňe nadať Počet negakí, ale len w ſamotních, a ſproſtích Numeroch, abí Počet, ku Počtu dodali. R. P. $\begin{array}{r}1\\ \underline{2}\\ 3\end{array}$ — $\begin{array}{r}3\\ \underline{4}\\ 7\end{array}$ — $\begin{array}{r}5\\ \underline{6}\\ 11\end{array}$ a tak dáleg. Po Čaſe ňech ſpoguge: dwa, ai tri Počti. R. P. $\begin{array}{r}123\\ \underline{456}\\ 579\end{array}$ — $\begin{array}{r}789\\ \underline{654}\\ 1443\end{array}$ — Na kterí Spôſob welice lechko Deti priprawí, ňelen ſamotné, ale ai zložené Počti dodáwať.

6. Welice ňerozumňe, a ňerozwážliwe čiňá ňekterí Normáliſtae, kterí Ďetom Millioni, ano až Stokrát Billioni predpiſugú, abí len ſwogu Pochwálu ziſkali, kďežto w takém krechkém Weku Ďiťa, aňi o Stom Numeru, doko.

Translation:

> *5. In the second half-year he (a teacher) can, within one or two weeks, assign an exercise, but only in digits in order to add number to number.*

E.g.

1	*3*	*5*	
2	*4*	*6*	
3	*7*	*11*	*etc.*

> *After a certain time a teacher should join from two to three numbers.*

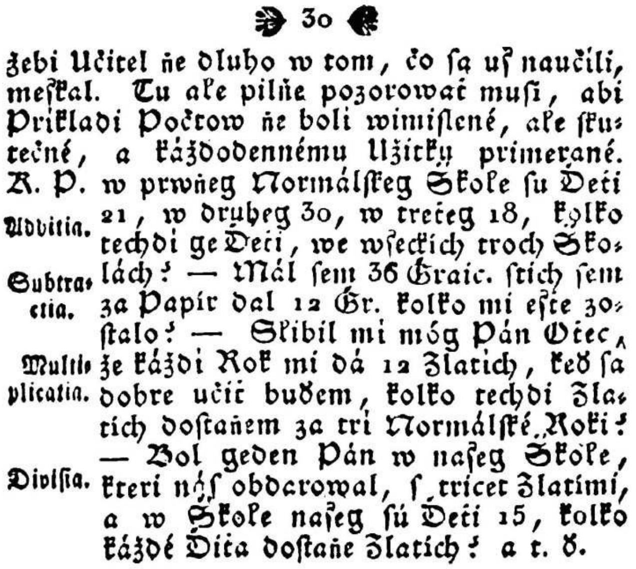
E.g.

123	*789*
456	*654*
579	*1443*

> *In this way he will very easily prepare children and they will know how to solve not only simple, but also composite problems.*

Figure 3. Excerpt from *Pedagogy* (p. 23)

Section 5, "On the art of counting" (on page 30), has interesting word assignments, which are supplemented with the following didactical note:



30

zebi Učitel ňe dluho w tom, čo ſa už naučili,
meſkal. Tu ale pilňe pozorowat muſi, abi
Priklady Počtow ňe boli wimiſlené, ale ſku-
tečné, a káždodennému Užitky primerané.
R. P. w prwňeg Normálſkeg Škole ſu Deti
Additio. 21, w druheg 30, w treteg 18, kolko
techdi ge Deti, we wſeckich troch Ško-
Subtra- lách? — Mál ſem 36 Graic. ſtich ſem
ctio. za Papír dal 12 Gr. kolko mi eſte zo-
ſtalo? — Slibil mi mój Pán Otec,
Multi- že káždi Rok mi dá 12 Zlatich, ked ſa
plicatio. dobre učit budem, kolko techdi Zla-
tich doſtaňem za tri Normálſké Roki?
— Bol geden Pán w naſeg Škole,
Diviſio. kteri náſ obdarowal, ſ tricet Zlatimi,
a w Škole naſeg ſú Deti 15, kolko
káždé Dita doſtaňe Zlatich? a t. d.

Translation:

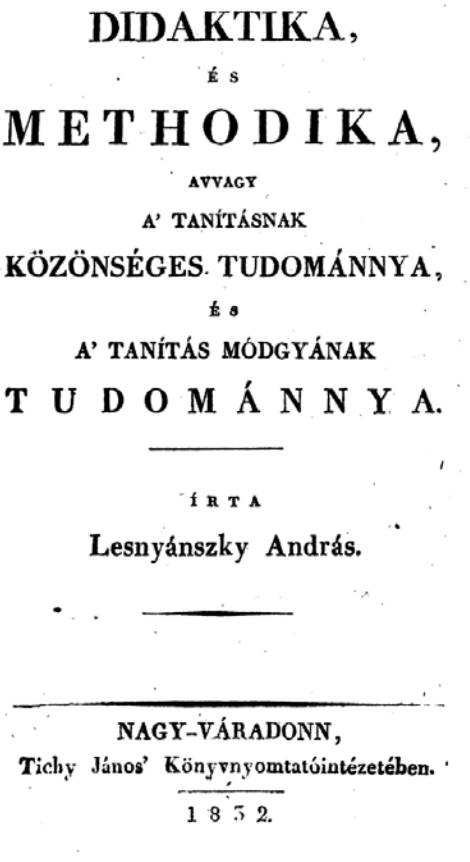
> *In the second grade of primary school children should repeat once again everything they learned in the first grade of primary school concerning the art of counting. It should not take too long, to avoid a delay. A teacher should pay attention to counting assignments, that they should be real, not made up, and suitable for everyday use. For example:*
>
> *Addition: In the first grade of primary school there are 21 children, in the second one 30, in the third one 18. How many children are there in all three grades?*
>
> *Subtraction: I had 36 kreutzers. I spent 12 kreutzers for paper. How much is left?*
>
> *Multiplication: My father promised me 12 guldens every year if I learnt well. How many guldens will I get for three years in primary school?*
>
> *Division: Our school was visited by a Sir who gave us 30 guldens. In our school there are 15 children. How many guldens will be given to each child? etc.*

Figure 4. Excerpt from *Pedagogy*, Section 5 (p. 30)

Pedagogy was written on 1820, but similar textbooks were also published in Ugria at that time. We can use one of these textbooks, *Didactics and Methodology*, to help us understand the context in which *Pedagogy* was written. *Didactics and Methodology* was written in 1832 and published in Oradea (see Lesnyánszky 1832). The author of this book was Lesnyánszky András (1795-1859), a Hungarian priest in a folk school in Oradea.

4. *Didactics and Methodology* by Lesnyánszky András

In comparison to *Pedagogy*, *Didactics and Methodology* has a more detailed description of the teaching of different subjects at primary level. It has more than 700 pages. The author used more materials and books from Austrian, Hungarian and German literature from the late 18th and early 19th century. The source most widely drawn from is *Didactics* by Joseph Weinkopf (see Weinkopf 1822).



DIDAKTIKA,
ÉS
METHODIKA,
AVVAGY
A' TANÍTÁSNAK
KÖZÖNSÉGES TUDOMÁNNYA,
ÉS
A' TANÍTÁS MÓDGYÁNAK
TUDOMÁNNYA.

ÍRTA
Lesnyánszky András.

NAGY-VÁRADONN,
Tichy János' Könyvnyomtatóintézetében.
1832.

Figure 5. Title page of the textbook, *Didactics and Methodology*

The first part of the book is devoted to general didactic questions. The next part to more concrete subjects (see Pukánszky 2012):

1. Understanding of surroundings
2. Knowledge of the human body and soul
3. Communication in society
4. Reading, writing and counting
5. Christian faith and moral education
6. History of the country.

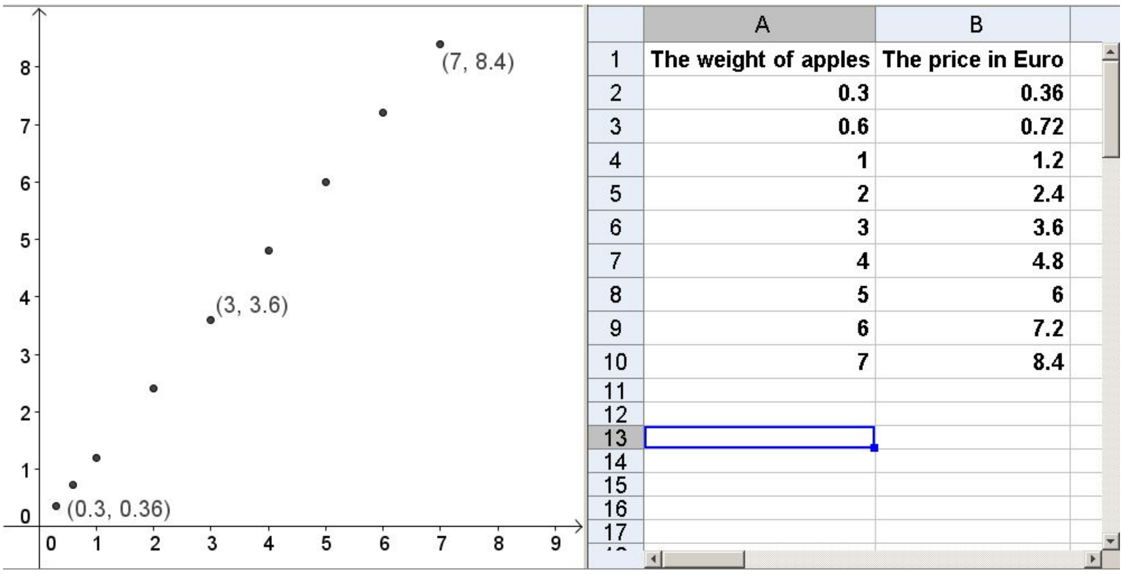
5. Mathematics Education in *Didactics and Methodology*

The mathematical section introduces a lot of separate and universal models for teaching different mathematical concepts and notions in arithmetic at the primary level. These models are important in the process of gaining knowledge in mathematics. According to Hejný and Littler (2006), the process of gaining knowledge in mathematics education is based on stages. It starts with motivation and at its core are two mental levels. The first leads from concrete knowledge to generic knowledge and the second from generic to abstract knowledge. Once knowledge is permanently acquired, the student moves to the stage of crystallisation, i.e. new knowledge is inserted into the already existing mathematical structure. The whole process consists of the following stages:

1. motivation
2. isolated models
3. generic model(s)
4. abstract knowledge
5. crystallisation
6. automation

Motivation is the tension which occurs in a person's mind as a result of the discrepancy between the existing and desired states of knowledge. The discrepancy comes from the difference between “I do not know” and “I need to know”, or “I cannot do that” and “I want to be able to do that”, sometimes from other needs and discrepancies too.

The pupils' experiences concerning some mathematical notion can be used as an isolated model, the next stage in the process. For example, the weight of apples according the cost of apples can be represented in the following table and graph (where 1kg apples is 1,20 Euro). In this case we can represent a function as a set of isolated points (see Figure 6).



	A	B
1	The weight of apples	The price in Euro
2	0.3	0.36
3	0.6	0.72
4	1	1.2
5	2	2.4
6	3	3.6
7	4	4.8
8	5	6
9	6	7.2
10	7	8.4
11		
12		
13		
14		
15		
16		
17		

Figure 6. The dependence of the price of apples on the weight of apples

This is similar to the way old Babylonian mathematicians studied the position of the planets and represented their movements with a set of isolated points (pictures of the positions of the planets). In this stage (isolated models), the pupils in the school can measure variables such as temperature, the height of a river in time and so on.

In the third stage (generic models) we can, for example, try to find the curve that is created by joining the points and can lead to the graph of the function, which represents the concrete relationship.

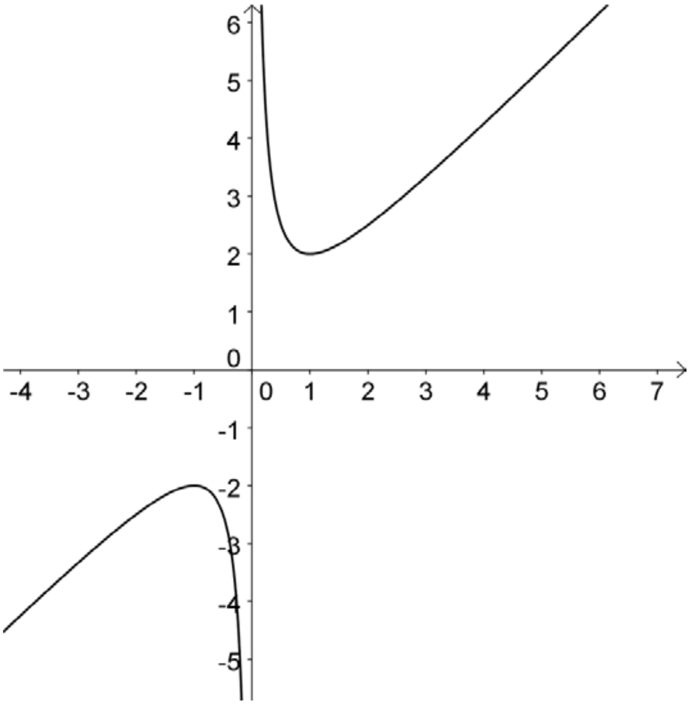
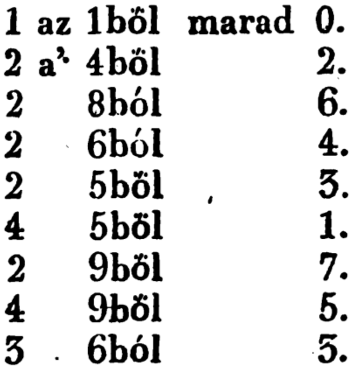


Figure 7. Example of the function, which is not continuous

In this way, abstract knowledge (stage 4) can be the definition of the function, which we received in a spontaneous and natural way from the previous isolated and generic models. Crystallisation (stage 5) and automation (stage 6) work with concrete functions and also with functions which are not continuous and have more complicated shapes.

We can see separate and universal models in Lesnyánszky's *Didactics and Methodology* in his explanation of subtraction at primary level (1832, 349). There he recommends using beans. We can, for example, take 9 beans in our right hand and then put 8 of those beans in our left hand. The teacher then asks the following question: How many beans do you now have in your right hand? The answer is one. The second question is: How many beans did you have in your right hand before? The answer is nine. The third question is: How many beans are in your left hand? The answer is eight.

The teacher can repeat this activity with other objects and can say to the pupils, if he puts nine objects in his right hand and he puts eight of them in his left hand, one object will always remain in his right hand. When pupils understand this rule, they understand that: 9 – 8 = 1. Lesnyánszky recommended preparing following table:



1	az	1ből	marad	0.
2	a'	4ből		2.
2		8ból		6.
2		6ból		4.
2		5ből		3.
4		5ből		1.
2		9ből		7.
4		9ből		5.
3		6ból		3.

Figure 8. *Didactics and Methodology* (p. 349)

1 – 1 = 0
4 – 2 = 2
8 – 2 = 6
6 – 2 = 4
5 – 2 = 3
5 – 4 = 1
9 – 2 = 7
9 – 4 = 5
6 – 3 = 3

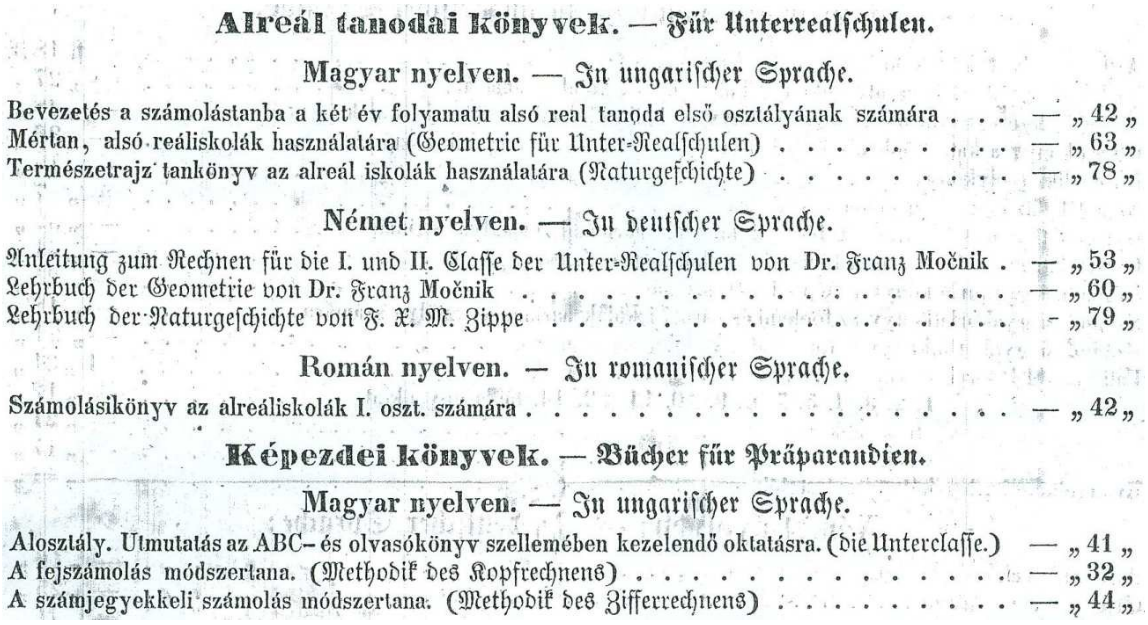
Table 1. Examples for the Figure 8

In Figure 8, the central column shows the numbers of objects in the right hand at the beginning. The left column gives the numbers of objects taken into the left hand. The right column shows the number of objects, which remain in the right hand. The examples in Table 1 give the corresponding mathematical equations for the objects in Figure 8. This table is used also for explaining of examples 10 – 10, 40 – 20, etc.

5. Franz Močnik's mathematics textbooks

In the second half of the 19th century the mathematics textbooks used in the Teachers' Institute in Spišská Kapitula were written by Franz Močnik (1814-1892). At that time Franz Močnik wrote mathematical textbooks for the whole Austro-Hungarian monarchy. Originally, textbooks were written in German, but most of them were translated to all of the languages used by the monarchy.

According to Hladnik (1997), these textbooks were translated into 13 different languages. We can see that these textbooks were used at the Teachers' Institute in Spišská Kapitula from Figure 9, which shows the attachment to a letter from 1862 detailing possible textbooks for use.



Alreál tanodai könyvek. — Für Unterrealſchulen.

Magyar nyelven. — In ungariſcher Sprache.

Bevezetés a számolástanba a két év folyamatu alsó real tanoda első osztályának számára . . . — „ 42 „
Mértan, alsó reáliskolák használatára (Geometrie für Unter-Realſchulen) . . . . — „ 63 „
Természetrajz tankönyv az alreál iskolák használatára (Naturgeſchichte) . . . . — „ 78 „

Német nyelven. — In deutſcher Sprache.

Anleitung zum Rechnen für die I. und II. Claſſe der Unter-Realſchulen von Dr. Franz Močnik . — „ 53 „
Lehrbuch der Geometrie von Dr. Franz Močnik . . . . — „ 60 „
Lehrbuch der Naturgeſchichte von F. X. M. Zippe . . . . - „ 79 „

Román nyelven. — In romaniſcher Sprache.

Számolásikönyv az alreáliskolák I. oszt. számára . . . . — „ 42 „

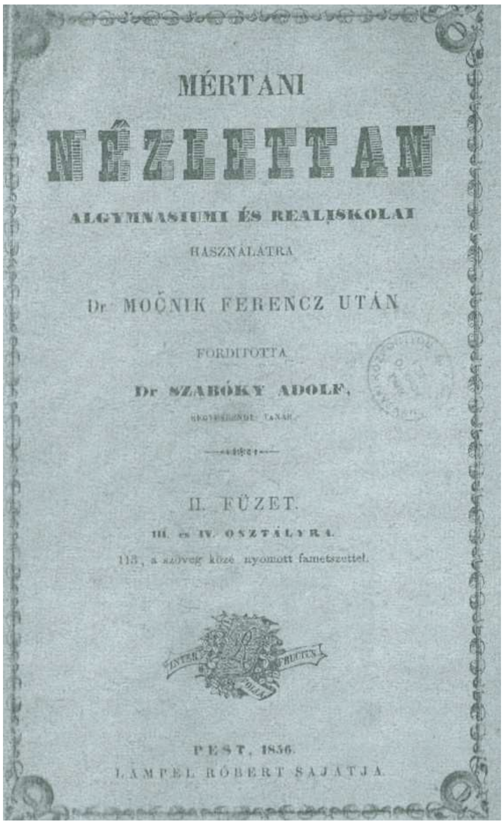
Képezdei könyvek. — Bücher für Präparandien.

Magyar nyelven. — In ungariſcher Sprache.

Alosztály. Utmutatás az ABC- és olvasókönyv szellemében kezelendő oktatásra. (die Unterclaſſe.) — „ 41 „
A fejszámolás módszertana. (Methodik des Kopfrechnens) . . . . — „ 32 „
A számjegyekkeli számolás módszertana. (Methodik des Zifferrechnens) . . . . — „ 44 „

Figure 9. Part of an attachment to a letter, recommending which textbooks to buy for the Teachers' Institute in Spišská Kapitula in 1862 (State Archive in Levoča)

We will now turn to two examples from a Močnik textbook on visual geometry. This textbook was a Hungarian translation and was published in Budapest in 1856 (see Figure 10 and Močnik & Szabóky 1856).



MÉRTANI
NÉZLETTAN
ALGYMNASIUMI ÉS REALISKOLAI
HASZNÁLATRA
Dr MOČNIK FERENCZ UTÁN
FORDITOTTA
Dr SZABÓKY ADOLF,

II. FÜZET.
III. és IV. OSZTÁLYRA.
113, a szöveg közé nyomott fametszettel.

PEST, 1856.
LAMPEL RÓBERT SAJÁTJA.

Figure 10. Textbook Močnik & Szabóky: *Mértani nézlettan* (Visual Geometry)

> *Example 1. Construct three circles with the radii m, n, p where each circle touches the other from the outside.*
>
> *Solution: We construct the triangle ABC with sides AB = m+n, AC = m+p and BC = n+p Now we construct the circle with centre A and radius m, the circle with centre B and radius n and the circle with centre C and radius p. The circles fulfil the objective of the task* (see also Figure 11)

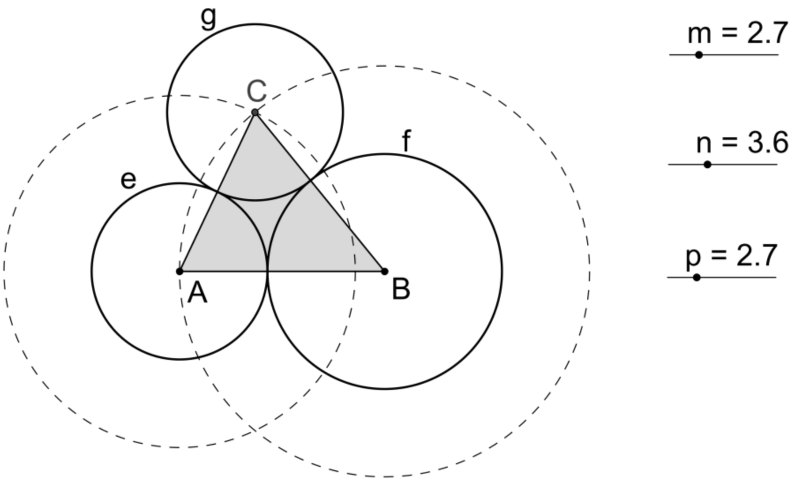


Figure 11. Construction of Example 1 prepared in open source software, *GeoGebra*

Example 2. We have made three circles of the same size, e, f, g, which touch each other outside. We must now circumscribe a fourth circle k, which the previous three circles are inside of.

Solution: Example 2 is a continuation of example 1 and has a logical connection to it. First we construct three circles of the same size, e, f, g in a similar way, to example 1. The circle k has its centre in the orthocentre of the triangle ABC (see also Figure 12).

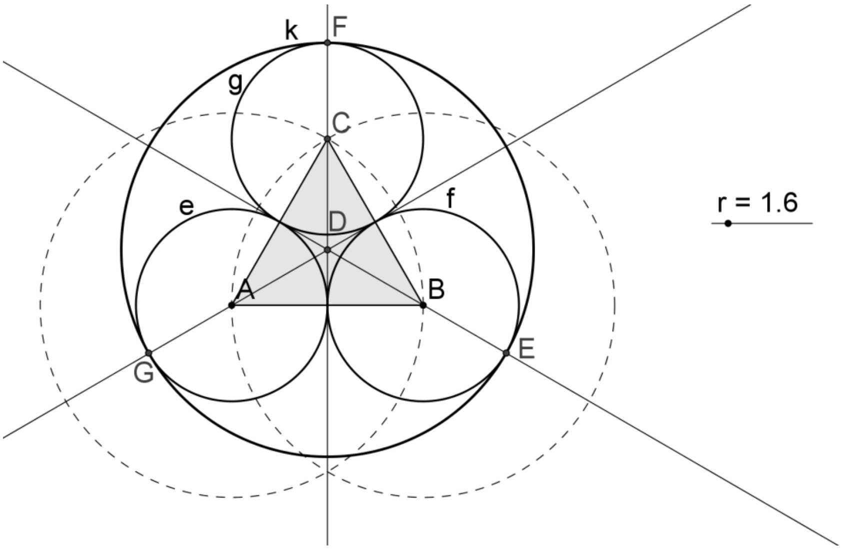


Figure 12. Construction of Example 2 prepared in open source software, *GeoGebra*

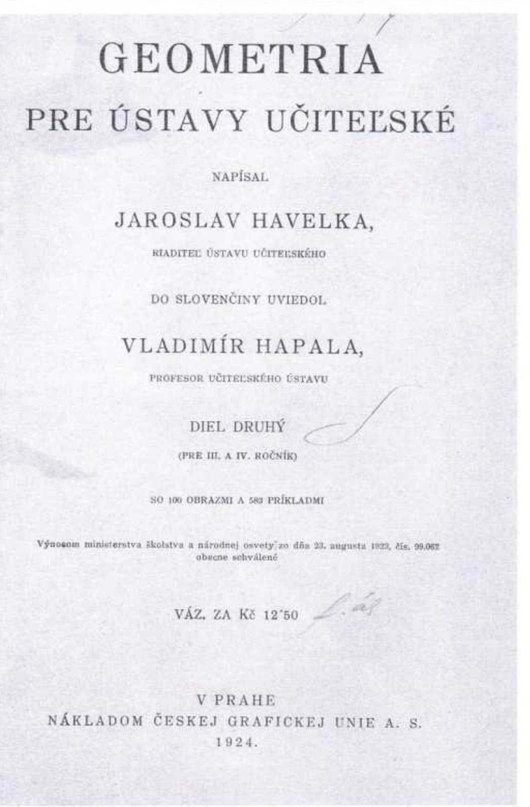
6. Jaroslav Havelka and Václav Posejpal textbooks

Owing to the ideology in former Czechoslovakia after the Second World War, the government put pressure on institutions founded or subscribed to by the Church. The Teachers' Institute in Spišská Kapitula was not exempt. After nationalisation in 1944, the pressure against it grew. It reached its climax in 1949 with the abolition of the institution. The fate of the written relics – the archives and library – was unclear during this period of time.

Part of the Teachers' Institute archive was moved to a school in Levoča, and it was subsequently moved to the State Archives in Levoča. Today it is known as the Archive Fund of the Teachers' Academy in Spišská Kapitula. There is also an Archive Fund of the Teachers' academy in Spišská Kapitula and a Library of the Teachers' Academy in Spišská Kapitula, within the Bishops' Archive of Spiš, in Spišské Podhradie. Both of the funds were established by the delimitation of the remains of documents from the Bishop Archive Fund of Spiš, by collection activities organised by the diocese of Spiš on the occasion of the 180th anniversary of the foundation of the Teachers' Institute. Apart from these three collections, it is possible that more relics from the Teachers' Institute are located in other archives and libraries in Slovakia or outside the country.

In the Bishops' Archive of Spiš in Spišské Podhradie there are two textbooks written by Czech authors, which were translated into Slovak. These textbooks were published at the time of the first Czechoslovakian republic.

First is the textbook of Jaroslav Havelka: *Geometria pre ústavy učiteľské* (*Geometry for Teachers´ Institutes*). This textbook is in the Library of the Teachers' Academy in Spišská Kapitula and we have Slovak translations of this textbook, which were made by Vladimír Hapala (see Figure 13 and Havelka 1924).



GEOMETRIA
PRE ÚSTAVY UČITEĽSKÉ

NAPÍSAL

JAROSLAV HAVELKA,
RIADITEĽ ÚSTAVU UČITEĽSKÉHO

DO SLOVENČINY UVIEDOL

VLADIMÍR HAPALA,
PROFESOR UČITEĽSKÉHO ÚSTAVU

DIEL DRUHÝ
(PRE III. A IV. ROČNÍK)

SO 100 OBRAZMI A 583 PRÍKLADMI

Výnosom ministerstva školstva a národnej osvety zo dňa 23. augusta 1923, čís. 99.062
obecne schválené

VÁZ. ZA Kč 12'50

V PRAHE
NÁKLADOM ČESKEJ GRAFICKEJ UNIE A. S.
1924.

Figure 13. The title page of slovak translation of Geometry for Teachers´ Institutes (1924)

We chose the following example from the book:

> *Prove that equilateral triangle inscribed within the circle has the same area as half of the equilateral hexagon inscribed within the same circle.*
>
> *Solution: If we have a circle with the centre* S *and radius* r, *we can use for the equilateral triangle* ABC *inscribed in this circle, following the expression for the area of a triangle* ABC (see Figure 14):

$$S_1 = \frac{1}{2}r^2 \cdot \sin 120° = \frac{1}{2}r^2 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}r^2$$

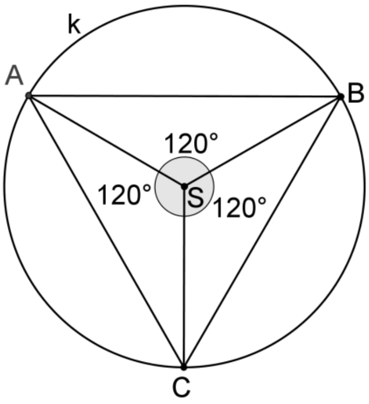


Figure 14. Equilateral triangle ABC

The area *S* of the whole triangle is

$$S_1 = 3S_1 = 3 \cdot \frac{\sqrt{3}}{4} r^2 = \frac{3\sqrt{3}}{4} r^2.$$

If we have one half of the equilateral hexagon - quadrilateral KLMN (see Figure 15), we can make the same algorithm as for triangle ABC.

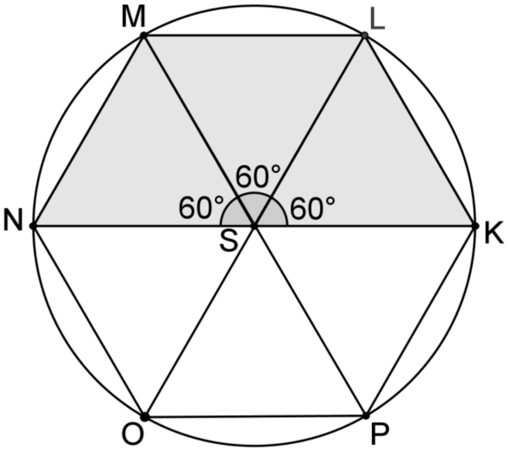


Figure 15. The half of the equilateral hexagon KLMN

The area of triangle KLS is:

$$S_2 = \frac{1}{2} r^2 \cdot \sin 60° = \frac{1}{2} r^2 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} r^2$$

The area, S_3, of the whole quadrilateral KLMN is:

$$S_3 = 3S_2 = 3 \cdot \frac{\sqrt{3}}{4} r^2 = \frac{3\sqrt{3}}{4} r^2.$$

We conclude that $S = S_3$.

Now we will continue with two examples from the textbook of Václav Posejpal: *Aritmetika pre ústavy učiteľské* (*Arithmetic for Teachers´ Institutes*, see Posejpal 1926). This textbook is also from the Library of the Teachers' Academy in Spišská Kapitula and we have Slovak translations of this textbook, also made by Vladimír Hapala. We used the exercise book from this textbook.

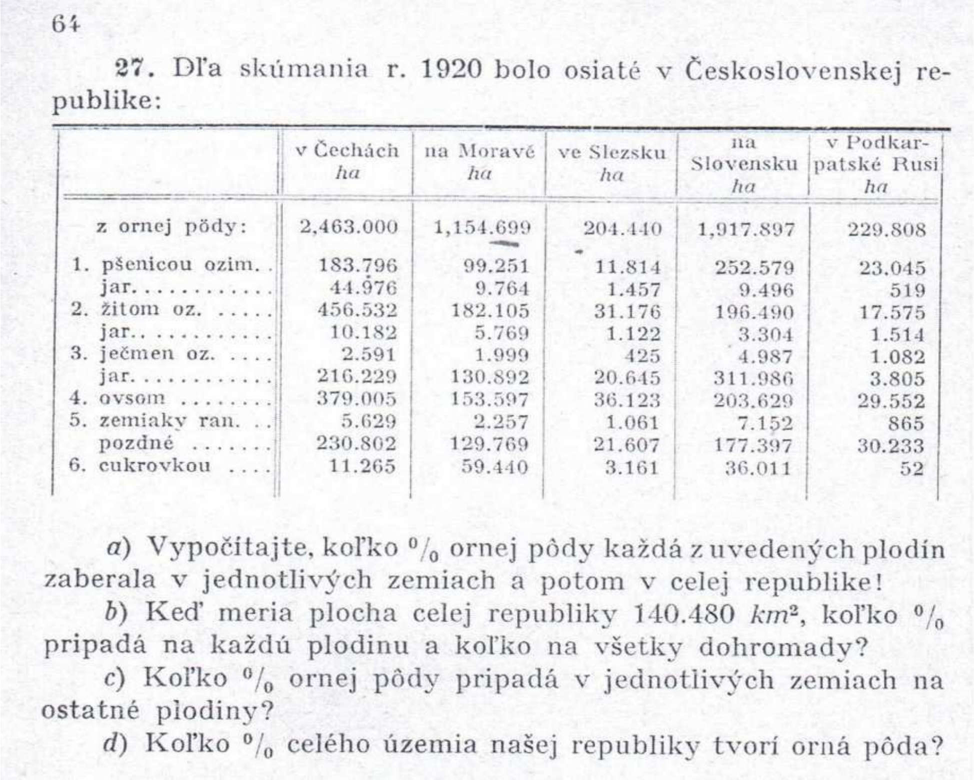
The following example (see Figure 16) shows that the data in some examples were taken from real-life situations in the Czechoslovakian republic between the two world wars. The example is about calculating percentages of agricultural earth devoted to different commodities: corn, barley, potatoes, oat, rye, and sugar-beet. The data are from different parts of the country: Slovakia, Bohemia, Moravia, Silesia and Carpatho-Ukraine. The questions in the example are as follows:

a) Calculate the percentage of each commodity in every part of the country and also in the whole country.

b) If the area of the whole country is 140,480 km^2, what percentage of the area is used for each commodity and what is percentage used by all commodities combined?

c) What percentage of agricultural earth remains in the whole of the country for other commodities?

d) What is the percentage of agricultural earth in the whole country?



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27. Dľa skúmania r. 1920 bolo osiaté v Československej republike:

	v Čechách *ha*	na Moravě *ha*	ve Slezsku *ha*	na Slovensku *ha*	v Podkarpatské Rusi *ha*
z ornej pödy:	2,463.000	1,154.699	204.440	1,917.897	229.808
1. pšenicou ozim. .	183.796	99.251	11.814	252.579	23.045
jar.	44.976	9.764	1.457	9.496	519
2. žitom oz.	456.532	182.105	31.176	196.490	17.575
jar..	10.182	5.769	1.122	3.304	1.514
3. ječmen oz.	2.591	1.999	425	4.987	1.082
jar.	216.229	130.892	20.645	311.986	3.805
4. ovsom	379.005	153.597	36.123	203.629	29.552
5. zemiaky ran. . .	5.629	2.257	1.061	7.152	865
pozdné	230.802	129.769	21.607	177.397	30.233
6. cukrovkou	11.265	59.440	3.161	36.011	52

a) Vypočítajte, koľko $^0/_0$ ornej pôdy každá z uvedených plodín zaberala v jednotlivých zemiach a potom v celej republike!

b) Keď meria plocha celej republiky 140.480 km^2, koľko $^0/_0$ pripadá na každú plodinu a koľko na všetky dohromady?

c) Koľko $^0/_0$ ornej pôdy pripadá v jednotlivých zemiach na ostatné plodiny?

d) Koľko $^0/_0$ celého územia našej republiky tvorí orná pôda?

Figure 16. Arithmetics Exercise book for the Teachers´ Institutes, p. 64

Here we present the solution for this exercise using a quadratic equation with the real parameter *b* from this textbook:

$$3x^2 - (b-9)x - 3b = 0 \quad (1)$$

The discriminant of this equation is:

$$D = (b-9)^2 + 36b = b^2 - 18b + 81 + 36b =$$

$$= b^2 + 18b + 81 = (b+9)^2.$$

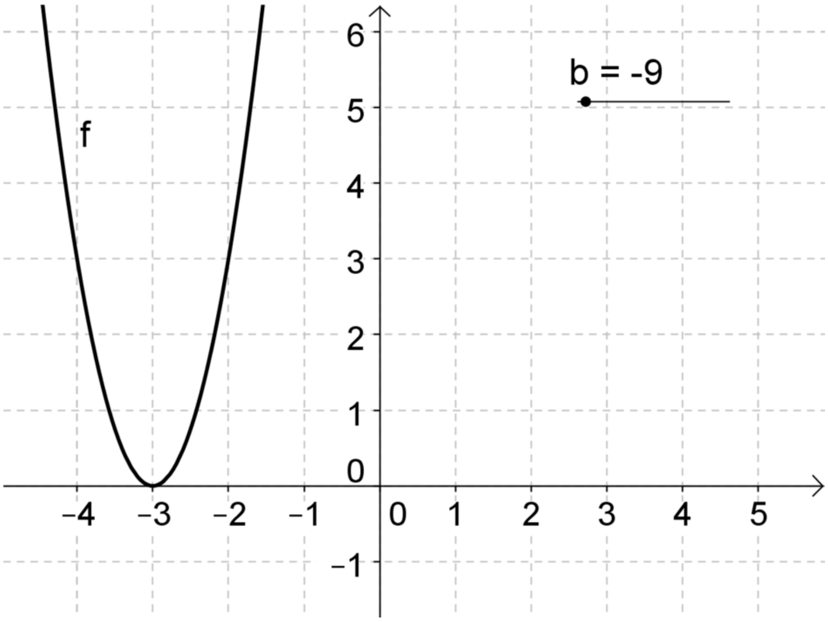

Figure 17. Graph of the function $\mathrm{f}(\mathrm{x}) = 3\mathrm{x}^2 - (\mathrm{b} - 9)\mathrm{x} - 3\mathrm{b}$ in the case $\mathrm{b} = -9$.

If $b \neq -9$, then $D > 0$ and equation (1) has two solutions:

$$x_{1,2} = \frac{(b-9) \pm |b+9|}{6}.$$

Hence

$$x_1 = \frac{b}{3}, x_2 = -3.$$

If $b = -9$, then $D = 0$ and equation (1) has one solution: $x = -3$.

This equation has also a geometrical interpretation. Every parabola,

$$y = 3x^2 - (b-9)x - 3b$$

must obtain the point [-3,0] (see also Figure 17).

7. Conclusions

The significance of the Teachers' Institute in Spišská Kapitula lies in the fact that it was the first of its kind in Central Europe. Although it was not one of the institutes with large numbers of scholars, its creation, curriculum and rules became the model for the establishment of all other pedagogical institutions devoted to educators in Central Europe. This first and oldest institute for primary school teachers on the Slovak territory included education in the Slovak language in its curriculum throughout its entire history, right after it was founded, that is, several decades before official attempts for national language codification were made. The Teachers' Institute is an extraordinary and suitable object of investigation for several branches of didactics in different subjects. There are textbooks in the State Archive in Levoča and the Bishops' archive in Spišská Kapitula, not only in mathematics, but also in other subjects (see Kopáčová 2012 and Albert 2012).

In future research we will be interested to analyse the content of different historical mathematical textbooks. We believe that examples from these textbooks are suitable for use in teaching future primary school teachers.

We celebrate in the year 2014 the 200th anniversary of the birth of Slovenian author of mathematical textbooks, Franz Močnik.

Researching textbooks written by this author gives us the opportunity to broaden our scope to the international landscape, because his texts were translated into 14 languages. Močnik was an important figure in mathematics education during the rule of the Austro-Hungarian monarchy in the 19th century.

Acknowledgement

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References

Albert, B. G. (2012). Educational scientific relevance of textbook revision research. *Hungarian Educational Research Journal, 2*(3) Retrieved from: http://herj.hu/2012/11/gabor-b-albert-educational-scientific-relevance-of-textbook-revision-research/

Gejdoš, M. (2007). Explanácia Pedagogiky Slovenskej pre Triviálne školy Spišského biskupstva podľa Juraja Paleša. In: *Týždeň európskej vedy na PF KU v Ružomberku. Zborník* *prednášok z týždňa európskej vedy* (pp. 34-46). Ružomberok: Pedagogická fakulta KU.

Havelka, J. (1924). *Geometria pre ústavy učiteľské. Diel druhý.* Praha: Česká grafická únia.

Hejný, M. & Littler, G. (2006). Transmissive and constructivist approaches to teaching. In Hejný, M., Jirotoková D. (Eds.) *Creative Teaching in Mathematics* (pp. 11-34). Praha: Univerzita Karlova.

Hladnik M. (1997). Franc Močnik, kratek oris življenja in dela. In Razpet, M. (Ed.) *Po stopinjah dr. Franca Močnika*. Idrija:Bogataj.

Kopáčová J. (2012). *Vývoj učebníc Prírodovedy na Slovensku*. Ružomberok : Verbum – vydavateľstvo Katolíckej univerzity v Ružomberku.

Močnik, F., & Szabóky, A. (1856). *Mértani nézlettan*. Budapest: Lampel Róbert Sajátja.

Olejník, V. (2007). Učiteľský ústav v Spišskej Kapitule. In Lukáč E., Michalička V. (Eds.) *História najstarších učiteľských ústavov na Slovensku*. Prešov: Filozofická fakulta Prešovskej univerzity (p. 225-277).

Páleš, J. (1820). *PAEDAGOGIA Slowenská pre Triwiálske Školi Biskúpstwa Spíšského spisaná* skrz *G. P.* Levoča: Ján Werthmüller.

Posejpal, V. (1926). *Aritmetika pre ústavy učiteľské*. Praha: Česká grafická únia.

Pukánszky B. (2012). Két tanítoi kézikönyv a múlt század első feléből. Retrieved from: http://www.pukanszky.hu/kez.htm

Pyrker J. L. (1996). *Mein Leben 1772-1847*. Wien: Aladar Paul Czigler.

Lesnyánszky A. (1832). *Didaktika és Methodika*. Oradea (Nagy – Várad): Tichy János Könyvnyomtató Intézet.

Weinkopf J. (1822). *Didaktik und Methodik*. Wien: Wallishauser.

Biographical note

Ján Gunčaga is Associate Professor of Mathematics Education at the Faculty of Education at the Catholic University in Ružomberok. His masters focused on the teaching of mathematics and physics at secondary level and was completed at the Faculty of Mathematics and Physics, Comenius University in Bratislava (1997). He completed his doctorate in Didactics of Mathematics in the Faculty of Science at Constantine the Philosopher University in Nitra (2004). His current research interests orient towards mathematics education at the primary and secondary level, the history of education and using ICT in educational processes.

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